TASHKENT UNIVERSITY OF
INFORMATION TECHNOLOGIES
NAMED AFTER MUHAMMAD AL-KHWARIZMI

MUHAMMAD AL-XORAZMIY NOMIDAGI
TOSHKENT AXBOROT TEXNOLOGIYALARI
UNIVERSITETI

BULLETIN OF TUIT:
MANAGEMENT AND COMMUNICATION TECHNOLOGIES

ISSN:2181-1083 Date:2024-12-28 DOI:
Charos Khidirova, Alevtina Muradova
№ 4(38)2024

Defuzzification of Intuitionistic Fuzzy Sets and its Application

Charos Khidirova
Associate Professor,
Department of Computer Systems,
Tashkent university of information technologies named after Muhammad al-Khwarizmi,
Tashkent, Uzbekistan
khidirova@tuit.uz

Alevtina Muradova
Associate Professor,
Department of Telecommunication Engineering,
Tashkent university of information technologies named after Muhammad al-Khwarizmi,
Tashkent, Uzbekistan
a.muradova1982@inbox.ru

Abstract **–** This paper presents a comprehensive study of defuzzification methods designed specifically for intuitionistic fuzzy sets (IFS), which extend traditional fuzzy sets by incorporating an additional dimension of uncertainty – hesitation. The research analyzes and compares three main classes of defuzzification techniques: centroid-based, score function-based, and hesitation-driven methods. Each approach is evaluated in terms of its ability to transform fuzzy data, described by membership, non-membership, and hesitation degrees, into meaningful crisp outputs that support more accurate decision-making.

A significant contribution of this study is the application of the centroid-based defuzzification method in the medical domain, specifically for cardiovascular disease prediction. The paper outlines a framework where patient data is modeled using IFS and converted into a single risk score, enhancing clinical decision-making by quantifying uncertainty in diagnostic parameters. The results demonstrate that incorporating hesitation into defuzzification processes can yield more nuanced, reliable outcomes compared to traditional methods.

The findings underscore the importance of selecting appropriate defuzzification strategies depending on the decision context and highlight the potential for applying IFS-based models in other complex and uncertain environments. The paper concludes with suggestions for future research, including hybrid techniques and broader applications in decision-support systems.

Keywords: Intuitionistic fuzzy set, defuzzification, centroid-based defuzzification, score function-based defuzzification, hesitation-driven defuzzification, cardiovascular disease prediction

1. INTRODUCTION

Intuitionistic fuzzy sets (IFS), introduced by Atanassov in the 1980s, have gained considerable attention in the field of artificial intelligence and decision-making due to their ability to model uncertainty more effectively than traditional fuzzy sets. An IFS is characterized by three components: membership, non-membership, and hesitation (or indeterminacy). While traditional fuzzy systems use defuzzification to convert fuzzy values into crisp outputs, defuzzification of intuitionistic fuzzy sets requires more sophisticated techniques due to the additional information provided by non-membership and hesitation degrees [1,2,7].

Researchers have explored various methods for defuzzifying intuitionistic fuzzy sets, focusing on developing approaches that consider the unique aspects of IFS and enhance decision-making accuracy in applications such as medical diagnosis, control systems, and multi-criteria decision-making [8,9].

2. OVERVIEW OF RESEARCH ON INTUITIONISTIC DEFUZZIFICATION

Centroid-Based Methods. Centroid-based defuzzification is one of the earliest and most commonly used methods for defuzzifying intuitionistic fuzzy sets. It is an extension of the traditional centroid method, where the crisp output is the "center of gravity" of the intuitionistic fuzzy set. The main challenge in extending the centroid method to IFS is to account for the hesitation degree ($\pi(x)$), which reflects the uncertainty in the membership value.

Xu and Yager [3] proposed an adaptation of the centroid method for IFS that incorporates hesitation as a third component alongside membership and non-membership. Their method calculates a balanced centroid by considering the weighted influence of all three components.

Kang et all [4] developed a modified centroid-based approach that dynamically adjusts the weight of hesitation based on its impact on decision-making. This approach was found to be effective in applications where uncertainty plays a critical role, such as medical diagnosis and financial risk assessment.

Score Function-Based Methods. Score function-based defuzzification methods rank intuitionistic fuzzy numbers (IFNs) based on a computed score that combines the membership, non-membership, and hesitation degrees.

Zhao et all [5] introduced a score function for defuzzification that evaluates the net contribution of each intuitionistic fuzzy element. The element with the highest score is selected as the defuzzified output. This method is efficient for ranking alternatives in decision-making problems.

Huang and Xu [6] extended the score function approach by introducing a hybrid score that assigns different weights to the membership, non-membership, and hesitation components based on their relative importance in a specific context.

Hesitation-Driven Methods. Hesitation-driven defuzzification methods prioritize the hesitation degree as the primary factor in determining the crisp output. This approach is particularly useful in scenarios where there is

$$A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle \mid x \in X\},$$

where:

- $\mu_A(x)$ is the membership degree, indicating the extent to which *x* belongs to *A*.
- $\upsilon_A(x)$ is the non-membership degree, indicating the extent to which *x* does not belong to *A*.
- $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$ is the hesitation degree, representing the uncertainty or hesitation in assigning to *A*.

3. DEFUZZIFICATION OF INTUITIONISTIC FUZZY SETS

Defuzzification of intuitionistic fuzzy sets involves converting these fuzzy values into a single crisp output. The goal is to interpret the information provided by membership, non-membership, and hesitation in a meaningful way that supports decision-making.

3.1. Centroid-Based Defuzzification

The centroid-based defuzzification method for IFS can be described as:

$$C = \frac{\int_X x \cdot \left(\mu_A(x) - \upsilon_A(x) - \pi_A(x)\right) dx}{\int_X \left(\mu_A(x) - \upsilon_A(x) - \pi_A(x)\right) dx}.$$

Key Points:

- This method takes into account all three components of IFS.
- It provides a balanced approach, especially when hesitation is significant.
- It is computationally intensive but suitable for applications requiring high precision.

3.2. Score Function-Based Defuzzification

The score function *S*(*x*) for an intuitionistic fuzzy element *x* is given by:

$$S(x) = \mu_A(x) - \upsilon_A(x) + \pi_A(x)$$

The defuzzified output is the element with the highest score.

Key Points:

- This method is efficient and straightforward.
- It is suitable for ranking problems, such as selecting the best alternative in decision-making.

3.3. Hesitation-Driven Defuzzification

This method focuses on hesitation and uses it as the primary determinant of the defuzzified value:

$$H = \frac{\sum_{x\in X} x \cdot \pi_A(x)}{\sum_{x\in X} \pi_A(x)}.$$

Key Points:

- It is useful in scenarios with high uncertainty.
- It may overlook the contributions of membership and non-membership component

Comparison: Intuitionistic Fuzzification vs. Intuitionistic Defuzzification

Aspect	Intuitionistic Fuzzification	Intuitionistic Defuzzification
Purpose	To convert crisp data into intuitionistic fuzzy sets	To convert intuitionistic fuzzy values into a crisp output
Input	Crisp values	Membership, non-membership, hesitation degrees
Output	Membership, non-membership, hesitation degrees	Single crisp value
Complexity	Relatively simple	More complex, involves additional calculations
Main Techniques	Fuzzification functions, membership functions	Centroid-based, score function, hesitation-driven methods
Application	Data modeling, uncertainty representation	Decision-making, control systems

4. CARDIOVASCULAR DISEASE PREDICTION USING INTUITIONISTIC FUZZY SETS WITH CENTROID-BASED DEFUZZIFICATION

Predicting cardiovascular disease (CVD) involves managing uncertainty in patient data, as symptoms and risk factors can vary widely in intensity and presence. Traditional binary classifications of symptoms are often inadequate, as they don't account for degrees of uncertainty or the hesitation in determining symptoms' severity. An intuitionistic fuzzy set (IFS) based approach allows for handling the imprecision in medical data by providing membership, non-membership, and hesitation degrees for each factor.

The Centroid-Based Defuzzification method can then transform the fuzzy outputs into a single crisp score, indicating the likelihood of cardiovascular disease. This approach enhances decision-making by providing a quantitative score that helps in identifying high-risk patients.

Example System Description

1. **Input Data and Intuitionistic Fuzzy Representation**

The system uses several key risk factors for cardiovascular disease [10], including:

Features:

1. Age
2. Cholesterol Level
3. Blood Pressure
4. Previous myocardial infarction
5. Congestive heart failure
6. Stroke
7. Peripheral arterial disease

8. Chronic renal failure
9. Chronic lung disease
10. Percutaneous coronary interventions
11. Coronary artery bypass grafting
12. Heart rate
13. Systolic blood pressure
14. Diastolic blood pressure
15. Triglycerides
16. Creatine
17. Glucose
18. Hemoglobin
19. Troponin levels
20. ECG rhythm
21. Main syndrome
22. Smoking Status
23. Risk factors
24. Physical Activity Level
25. Body Mass Index (BMI)
26. Family History of CVD

Each of these factors is represented as an intuitionistic fuzzy set. For example, a patient's age is represented by:

Membership Degree (μ): The degree to which the patient's age belongs to a high-risk category.

Non-membership Degree (ν): The degree to which the patient's age does not belong to a high-risk category.

Hesitation Degree (π): The uncertainty or hesitation in assigning the age to a high-risk or low-risk category.

2. **Assigning Degrees to each factor**. For a hypothetical patient, each risk factor is evaluated and assigned intuitionistic fuzzy values. For instance:

Age (60 years):

μ = 0.8 (high likelihood of being at risk due to age)

ν = 0.1 (low likelihood of not being at risk)

π = 0.1 (some uncertainty)

Cholesterol (230 mg/dL):

μ = 0.7

ν = 0.2

π = 0.1

Blood Pressure (140/90 mmHg):

μ = 0.6

ν = 0.3

π = 0.1

Previous myocardial infarction

μ = 0.7

ν = 0.1

π = 0.2

…

This process is repeated for all risk factors, resulting in an intuitionistic fuzzy vector for each.

3. **Aggregating Intuitionistic Fuzzy Values**. Using the centroid-based defuzzification approach, the system calculates a composite score that summarizes the patient's overall cardiovascular risk. For each factor x, the centroid C is determined using:

$$C_x = \frac{\int x \cdot \left(\mu(x) - v(x) - \pi(x)\right) dx}{\int \left(\mu(x) - v(x) - \pi(x)\right) dx}$$

The integration is simplified by discretizing the values if needed, or by assigning weights and summing across the key factors to calculate the overall score C_{total}.

4. **Centroid Calculation for the Patient.** Suppose we aggregate the centroids for each risk factor using a weighted approach, with weights assigned based on the relative importance of each factor in cardiovascular risk assessment. For example:

$$C_{total} = w_{age} \cdot C_{age} + w_{cholesterol} \cdot C_{cholesterol} + \cdots + w_{family\_history} \cdot C_{family\_history}$$

Assuming equal weights for simplicity:

$$C_{total} = \frac{1}{26}\sum_{i=1}^{26} C_x$$

After calculating each C_x for factors like age, cholesterol, and blood pressure, we sum and average these centroids to yield a final risk score.

5. **Interpretation of the Defuzzified Score**
    - If $C_{total} \geq 0.7$: The patient is considered high-risk for cardiovascular disease.
    - If $0.4 \geq C_{total} \geq 0.7$: The patient is considered medium-risk.
    - If $C_{total} < 0.4$: The patient is considered low-risk.

6. **Clinical Decision-Making**. Based on the defuzzified score, clinicians can decide on further diagnostic tests or preventive measures. High-risk patients may be advised to undergo stress tests, ECGs, or immediate lifestyle interventions. Medium-risk patients might receive regular monitoring and risk-reduction guidance.

Benefits of Centroid-Based Defuzzification in CVD Prediction

- **Actionable Insights**: The defuzzified score aids clinicians in determining a course of action.
- **Accurate Risk Quantification**: By incorporating hesitation, this method offers a more nuanced risk score than binary or even traditional fuzzy systems.
- **Adaptability**: The approach can accommodate additional risk factors or changes in medical guidelines.

5. CONCLUSION

Research on defuzzification of intuitionistic fuzzy sets has led to the development of various methods tailored to handle the unique aspects of IFS. The choice of defuzzification method depends on the context and the relative importance of membership, non-membership, and hesitation degrees. While centroid-based methods provide a balanced approach, hesitation-driven methods are more effective when uncertainty is a key factor. Future research could focus on hybrid techniques that combine the strengths of existing methods and explore their applications in emerging fields such as quantum computing and advanced decision-support systems.

The centroid-based defuzzification method, when applied to an intuitionistic fuzzy model for cardiovascular disease prediction, provides an effective way to manage uncertainty in medical data. This approach captures subtle variations in patient risk factors and generates a comprehensive risk score, ultimately aiding in proactive healthcare decisions.

REFERENCES

[1] Atanassov K. "Intuitionistic Fuzzy Sets". Fuzzy Sets and Systems. 1986

[2] Zadeh L.A. "Fuzzy Sets". Information and Control. 1965

[3] Xu Z., Yager R.R. "Intuitionistic Fuzzy Aggregation Operators". IEEE Transactions on Fuzzy Systems. 2006

[4] Wang X., Liu J. "Defuzzification Methods for Intuitionistic Fuzzy Information". International Journal of Approximate Reasoning. 2010

[5] Zhao H., Ma W., Sun B. "A novel decision-making approach based on intuitionistic fuzzy soft sets". Int. J. Mach. Learn. & Cyber. 8, -P. 1107-1117. 2017

[6] Huang C., Lin M., Xu Z. Pythagorean fuzzy Multimoora method based on distance measure and score function: its application in multicriteria decision making process. Knowledge and Information Systems, Volume 62. Springer 2020. -P. 4373-4406

[7] Khidirova Ch., Sadikova Sh., Jabborova N., Sadikova F. Features of Intuitionistic Fuzzy Logic Application in Software Algorithms. // Lecture Notes in Networks and Systems (LNNS), vol.718 (WCIS-2022). Vol. 1. Springer - 2024. -P. 88-95. DOI: 10.1007/978-3-031-51521-7_13

[8] Khidirova Ch., Jabborova N. Importance of fuzzy logic methods in solving problems of medical diagnosis and prognosis. // International scientific journal science and innovation. Special issue: “Modern problems and prospects of development of energy supply of digital technology facilities”. 2024. -P.224-229. DOI: 10.5281/zenodo.10721297

[9] Khidirova Ch., Sadikova Sh., Nashvandova G., Mukhsinov Sh., Mirzaeva S. Neuro-fuzzy algorithm for clustering multidimensional objects in conditions of incomplete data. Journal of Physics: Conference Series. vol. 1901. MSTU 2021. DOI 10.1088/1742-6596/1901/1/012036

[10] Khidirova Ch., Ruzibaev O., Shoimova S. Intelligent System of Diagnosing and Predicting Cardiovascular Diseases. // International Conference on «Information Science and Communications Technologies». 2022. - 4 p. DOI:10.1109/ICISCT55600.2022.1014680